Tool Tips

In the following pages, you will find descriptions of each of the tools that were pre-made for use in the *Exploring Calculus* activities. The steps for making these tools were included here rather than in the book so as not to detract from the mathematical focus of the activity.

For a full explanation of **Custom** tools, see *The Geometer's Sketchpad Reference Manual.*

Average Rate

Activity: "Plotting Average Rates of Change"

Document: **Step.gsp**

Given: Points A and B

Result: Calculation of the ratio of the change in y-coordinates to the change in x-coordinates between A and B and segment AB

1. Construct two points, A and B. Measure the x- and y-coordinates of each point.
2. Construct segment AB.
3. Calculate the average rate of change: $(y_B - y_A)/(x_B - x_A)$.
4. Select points A and B, segment AB, and the ratio $(y_B - y_A)/(x_B - x_A)$. Choose **Create New Tool** from **Custom** tools.

To have the tool use a different label for the calculation $(y_B - y_A)/(x_B - x_A)$, edit the label for the expression, and check Use Label in Custom Tools.

Plot Slope

Activity: "Plotting Average Rates of Change"

Document: **Step.gsp**

Given: Points A and B

Result: A segment between the points (x_A, *slope*(AB)) and (x_B, *slope*(AB)): in other words, a "step" that plots the slope of the segment, and segment AB.

1. Construct two points, A and B, and segment AB.
2. Measure the x- and y-coordinates of both points.
3. Calculate the slope: $(y_B - y_A)/(x_B - x_A)$. Label this measurement *slope(AB),* and check Use Label in Custom Tools.
4. Plot the points (x_A, *slope(AB)*) and (x_B, *slope(AB)*).
5. Construct a segment between these two plotted points.
6. Select points A and B, segment AB, and the segment you constructed in step 5. Choose **Create New Tool** from **Custom** tools.

You can also include the slope measurement in your tool.

Slope Tool

Activity: "Slope and Limits"

Document: **SlopeandLimit.gsp**

Given: Points *A* and *B*

Result: Segment *AB* and the slope of *AB*

1. Construct two points, *A* and *B,* and segment *AB.*
2. Select the segment and choose **Measure | Slope.**
3. Select points *A* and *B,* segment *AB,* and the slope measurement. Choose **Create New Tool** from **Custom** tools.

Including the segment in the tool is optional.

Slope + Line

Activity: "Slope and Limits"

Document: **SlopeandLimit.gsp**

Given: Points *A* and *B*

Result: Line *AB* and the slope of *AB*

1. Construct two points, *A* and *B,* and line *AB.*
2. Select the line and choose **Measure | Slope.**
3. Select points *A* and *B,* line *AB,* and the slope measurement. Choose **Create New Tool** from **Custom** tools.

Secant Line

Activity: "Derivatives and Transformations"

Document: **Transformations.gsp**

Given: Point *P*, function *f,* and measurement *h*

Result: A secant line through the points $(x_P, f(x_P))$ and $(x_P + h, f(x_P + h))$ and the slope of the line.

1. Create a point *P*, any function *f,* and any measurement *h.* (A slider works best for this tool.)
2. Measure the *x*-coordinate of point *P*.
3. Calculate $f(x_P)$, $x_P + h$, and $f(x_P + h)$.
4. Plot the points $(x_P, f(x_P))$ and $(x_P + h, f(x_P + h))$.
5. Construct the line through the two new points.
6. Calculate the slope of the line. Label it *SlopeSecant,* and check Use Label in Custom Tools.

7. Select point P, function f, measurement h, points $(x_P, f(x_P))$ and $(x_P + h,\ f(x_P + h))$, the line, and measurement *SlopeSecant.* Then choose **Create New Tool** from **Custom** tools.

PtSlopeLine

Activity: "Derivatives and Transformations"

Document: **Transformations.gsp**

Given: Point A and measurement *slope*

Result: A line through point A with the given measurement as the slope

1. Construct a point and any measurement (slider, parameter, or calculation). Label the point A and the measurement *slope*.
2. Measure the x- and y-coordinates of the point.
3. Plot the function $y_A + slope(x - x_A)$.
4. Select point A, measurement *slope*, the line, and the equation of the line, and choose **Create New Tool** from **Custom** tools.

Tangent Line

Activities: "Second Derivatives," "Newton's Method"

Documents: **SecondDerivatives.gsp, Newton.gsp**

Given: Point A and function f

Result: A line through point A with slope equal to the derivative of f at A

If point A is originally placed on the function plot of f, the tool will produce a tangent line at a random point on f. Originally plotting point A not on f allows you to choose the point.

1. Plot function f and construct point A not on f.
2. Measure the x-coordinate of point A.
3. Graph the derivative of function f, and calculate $f'(x_A)$.

You can construct a line with a slope of $f'(x_A)$ in several ways. Here is one approach:

4. Measure the y-coordinate of point A.
5. Calculate $f(x_A)$, $x_A + 1$, and $y_A + f'(x_A)$.
6. Plot the points $(x_A + 1, y_A + f'(x_A))$ and $(x_A, f(x_A))$ and construct the line between them.
7. Select point A, function f, the line you constructed in step 6, and the point $(x_A, f(x_A))$. Choose **Create New Tool** from **Custom** tools.

Alternatively:

4. Calculate $f(x_A)$ and plot the point $(x_A, f(x_A))$. Label this point *P*.
5. Plot the tangent line: $f(x_A) + f'(x_A)(x - x_A)$.
6. Construct point *Q* on this function plot and hide the function plot.
7. Construct the line through points *P* and *Q*.
8. Measure the slope of line *PQ*. Label this measurement *slope,* and check Use Label in Custom Tools.
9. Select point *A*, function *f,* the line you constructed in step 7, point *P*, and measurement *slope.* Choose **Create New Tool** from **Custom** tools.

Newton

Activity: "Newton's Method"

Document: **Newton.gsp**

Given: Point *A,* function *f,* and the *x*-axis

Result: A segment tangent to *f* at the point $(x_A, f(x_A))$, and a point at the intersection of this segment with the *x*-axis

When the tool is constructed for the first time as described in the activity "Newton's Method," point *A* should not be on the *x*-axis, so that the tool does not select a random point on the axis. By following the steps as written, you can choose the point at which to begin Newton's Method. However, when using the tool, the matched point should be placed on the *x*-axis.

Antiderivative Segment

Activity: "Step to the Antiderivative"

Document: **StepTangent.gsp**

Given: Point *R*, function *f,* and parameter *h*

Result: A segment with slope $f(x_R)$ and endpoints at $(x_R, f(x_R))$ and $(x_R + h, f(x_R + h))$.

The steps for constructing this tool are described in the activity.

The tool can be used to "stitch" together a series of segments that approximate a plot of the antiderivative of function *f.* The tool as built in the activity will match function *f* automatically. If a function *f* is not present in the sketch, you will need to match the tool with a function in addition to matching a point. Note that the tool will automatically match a function labeled *f* in the sketch—even if you do not wish to match this function. To avoid this outcome, give all functions labels other than *f.*

Anti-Segment2

Activity: "Step to the Antiderivative"

Document: **StepTangent.gsp**

Given: Points A, B, and C

Result: A segment with slope y_A, one endpoint at point (x_A, y_B), and the other at $(x_C, y_B + y_A(x_C - x_A))$

The steps for constructing this tool are described in the activity.

The tool can be used to "stitch" together a series of segments that approximate a plot of the antiderivative of a function defined by geometric shapes rather than by an algebraic expression. You can use it to plot approximate derivatives of piecewise defined functions.

Anti-Segment3

Activity: "Step to the Antiderivative"

Document: **StepTangent.gsp**

Given: Points A, B, and C

Result: A segment (and its endpoints) through the point (x_A, y_B) with slope y_A, tangent to an antiderivative of the function

You can build this tool on ExtraPage1 of **StepTangent.gsp.**

1. Construct point A on the function. Measure its x- and y-coordinates.
2. Construct an independent point B. Measure its y-coordinate.
3. Plot the point (x_A, y_B).
4. Define the function for the tangent line for the antiderivative through the point (x_A, y_B): $y_B + y_A(x - x_A)$. Label the function T, and check Use Label in Custom Tools.
5. To construct a small tangent segment, construct a second point, C, anywhere on the semicircle. Measure the x-coordinate of this point and calculate $T(x_C)$.
6. Plot the point $(x_C, T(x_C))$. Label this point B (you will have two points labeled B). Uncheck Show Label, and check Use Label in Custom Tools.
7. Construct the segment between (x_A, y_B) and $(x_C, T(x_C))$.
8. Select points A, C, B, both plotted points, and the segment. Choose **Create New Tool** from **Custom** tools. Name it **Anti-Segment3.** Check Show Script View.
9. In the tool's Script View, double-click Given point B and check Automatically Match Sketch Object.

This tool can now be used to construct an antiderivative of this and any other function with just the function's plot and a starting initial point for the antiderivative. Unlike the tool **Anti-Segment2**, this tool will string the antiderivative construction itself by using the right endpoint of the previously constructed segment, *but* only if the very first initial point *B*'s label is hidden.

To use the tool, go to ExtraPage2. There is a point in the plane that determines the initial y-value for the antiderivative. It is labeled *B* but that label is hidden—and must stay hidden for the tool to work. Choose **Anti-Segment3** from **Custom** tools. Construct two points on the first line segment. A segment with a slope equal to the y-value of the first point is created at the point (x_A, y_B). The spacing between the two points on the function plot determines the segment's length.

So that your constructed antiderivative is all one piece, match the first point with the last constructed point.

Left Rectangles

Activity: "Building Area"

Document: **BuildArea.gsp**

Given: Point *A*, function *f*, and parameter *h*

Result: A rectangle with width *h* and height $f(x_A)$. Point *A* is the lower left vertex of the rectangle. A point at $(x_A + h, y_A)$ is also constructed, allowing you to build another rectangle using that point as its lower left vertex.

When you use the tool, put point *A* on the x-axis.

1. Construct point *A* not on the x-axis, and measure its x- and y-coordinates.
2. Calculate $x_A + h$ and $f(x_A)$.
3. Plot the points $(x_A, f(x_A))$, $(x_A + h, f(x_A))$, and $(x_A + h, y_A)$.
4. Construct segments between point *A* and the three plotted points to form the rectangle.
5. Select point *A* and the three plotted points and choose **Construct | Quadrilateral Interior.**
6. Calculate the area of the rectangle: $h \cdot (f(x_A) - y_A)$. Label this measurement *Area* and check Use Label in Custom Tools.
7. Select point *A,* function *f,* parameter *h,* the quadrilateral interior, the segments you constructed in step 4, the point $(x_A + h, y_A)$, and measurement *Area.* Choose **Create New Tool** from **Custom** tools.
8. To automatch *f* and *h,* open the Script View. Double-click on *f* and *h,* and check Automatically Match Sketch Object.

Tool Tips (continued)

An alternative method for building this tool is described in the Activity Notes for “Building Area.” In that method, you create rectangles by matching a function, whereas in the steps above you create the rectangles by matching a point.

Right Rectangles

Activity: “Building Area”

Document: **BuildArea.gsp**

Given: Point A, function f, and parameter h

Result: A rectangle with width h and height $f(x_A + h)$. Point A is the lower left vertex of the rectangle. A point at $(x_A + h, y_A)$ is also constructed, allowing you to build another rectangle using that point as its lower left vertex.

When you use the tool, put point A on the x-axis.

1. Construct point A not on the x-axis, and measure its x- and y-coordinates.
2. Calculate $x_A + h$ and $f(x_A + h)$.
3. Plot the points $(x_A, f(x_A + h))$, $(x_A + h, f(x_A + h))$, and $(x_A + h, y_A)$.
4. Construct segments between point A and the three plotted points to form the rectangle.
5. Select point A and the three plotted points, and choose **Construct | Quadrilateral Interior.**
6. Calculate the area of the rectangle: $h \cdot (f(x_A + h) - y_A)$. Label this measurement *Area,* and check Use Label in Custom Tools.
7. Select point A, function f, parameter h, the quadrilateral interior, the segments you constructed in step 4, the point $(x_A + h, y_A)$, and measurement *Area.* Choose **Create New Tool** from **Custom** tools.
8. To automatch f and h, open the Script View. Double-click on f and h, and check Automatically Match Sketch Object.

An alternative method for building this tool is described in the Activity Notes for “Building Area.” In that method, you create rectangles by matching a function, whereas in the steps above you create the rectangles by matching a point.

Midpoint Rectangles

Activity: “Building Area”

Document: **BuildArea.gsp**

Given: Point A, function f, and parameter h

Result: A rectangle with width h and height $f(x_A + h/2)$. Point A is the lower left vertex of the rectangle. A point at $(x_A + h, y_A)$ is also constructed, allowing you to build another rectangle using that point as its lower left vertex.

When you use the tool, put point A on the x-axis.

1. Construct point A and measure its x- and y-coordinates.
2. Calculate $x_A + h/2$ and $f(x_A + h/2)$.
3. Plot the points $(x_A, f(x_A + h/2))$, $(x_A + h, f(x_A + h/2))$, and $(x_A + h, y_A)$.
4. Construct segments between point A and the three plotted points to form the rectangle.
5. Select point A and the three plotted points, and choose **Construct | Quadrilateral Interior.**
6. Calculate the area of the rectangle: $h \cdot (f(x_A + h/2) - y_A)$. Label this measurement *Area,* and check Use Label in Custom Tools.
7. Select point A, function f, parameter h, the quadrilateral interior, the segments you constructed in step 4, the point $(x_A + h, y_A)$, and measurement *Area.* Choose **Create New Tool** from **Custom** tools.
8. To automatch f and h, open the Script View. Double-click on f and h, and check Automatically Match Sketch Object.

Trapezoids

Activity: "Building Area"

Document: **BuildArea.gsp**

Given: Point A, function f, and parameter h

Result: A trapezoid with width h and bases with length $f(x_A)$ and $f(x_A + h)$. Point A is the lower left vertex of the trapezoid. A point at $(x_A + h, y_A)$ is also constructed, allowing you to build another trapezoid using that point as its lower left vertex.

When you use the tool, put point A on the x-axis.

1. Construct point A and measure its x- and y-coordinates.
2. Calculate $f(x_A)$, $x_A + h$, and $f(x_A + h)$.
3. Plot the points $(x_A, f(x_A + h))$, $(x_A + h, f(x_A + h))$, and $(x_A + h, y_A)$.
4. Construct segments between point A and the three plotted points to form the trapezoid.
5. Select point A and the three plotted points, and choose **Construct | Quadrilateral Interior.**

6. Calculate the area of the trapezoid: $h/2 \cdot (f(x_A + h) + f(x_A) - 2y_A)$. Label this measurement *Area,* and check Use Label in Custom Tools.

7. Select point *A,* function *f,* parameter *h,* the quadrilateral interior, the segments you constructed in step 5, the point $(x_A + h, y_A)$, and measurement *Area.* Choose **Create New Tool** from **Custom** tools.

8. To automatch *f* and *h,* open the Script View. Double-click on *f* and *h,* and check Automatically Match Sketch Object.

Trapezoid

Activity: "The Trapezoid Tool"

Document: **TrapezoidTool.gsp**

Given: Points *A* and *B,* and function *f*

Result: A trapezoid formed by points *A*, *B,* and the points at $(x_A, f(x_A))$ and $(x_A, f(x_B))$, and the area of this trapezoid

The steps for constructing this tool are described in the activity.

In the **Trapezoids** tool above, the width of the trapezoid is determined by a parameter. This tool allows you to determine the width of the trapezoid using two points that you match on the *x*-axis. The Activity Notes for "The Trapezoid Tool" include steps for creating a trapezoid tool that you can use without defining or matching a function.

Riemann Rectangles

Activity: "Area and Integrals"

Document: **AreaIntegral.gsp**

Given: Points *A* and *B,* and function *f*

Result: A rectangle with a height determined by the value of the function at a random point on the interval between x_A and x_B.

1. Construct two points, *A* and *B,* on the *x*-axis, and measure their *x*-coordinates.

2. Construct segment *AB.*

3. Construct point *i* on segment *AB* and measure its *x*-coordinate. Check Use Label in Custom Tools.

4. Calculate $f(x_i)$.

5. Plot the points $(x_i, f(x_i))$, $(x_A, f(x_i))$, and $(x_B, f(x_i))$.

6. Select points *A* and *B* and the two plotted points at $(x_A, f(x_i))$ and $(x_B, f(x_i))$, and choose **Construct | Quadrilateral Interior.**

7. Construct segments between the points *A*, *B*, and the two plotted points at $(x_A, f(x_i))$ and $(x_B, f(x_i))$.

8. Calculate $x_B - x_A$. Label this measurement *h*, and check Use Label in Custom Tools.

9. Calculate the area of the rectangle: $h \cdot f(x_i)$. Label this measurement *area*, and check Use Label in Custom Tools.

10. Select points *i* and $(x_i, f(x_i))$ and make them a different color than all the other points.

11. Select points *A*, *B*, function *f*, the segments you constructed in step 7, point *i*, point $(x_i, f(x_i))$, the quadrilateral interior, and measurement *area*. Choose **Create New Tool** from **Custom** tools.

12. To automatch *f*, open the Script View, double-click on Given function *f*, and check Automatically Match Sketch Object.

Plot Area with Trapezoids

Activity: "Plotting the Integral"

Document: **PlotIntegral.gsp**

Given: Point *A*, function *f*, and parameter *h*

Result: A trapezoid with width *h* between the plot of *f* and the *x*-axis, and a segment *AB*. The change in *y* between points *A* and *B* is equal to the area of the trapezoid, allowing you to use segment *AB* to create an approximate plot of the integral of $f(x)$.

1. Construct point *A* and measure its *x*- and *y*-coordinates.

2. Calculate $f(x_A)$, $x_A + h$, and $f(x_A + h)$.

3. Plot the points $(x_A, f(x_A))$ and $(x_A + h, f(x_A + h))$.

4. Calculate $x_A - x_A$. (This will give you a value of zero to use in the next step.) Label this *zero*, and check Use Label in Custom Tools.

5. Plot the points $(x_A, zero)$ and $(x_A + h, zero)$.

6. Select these four plotted points, and choose **Construct | Quadrilateral Interior**.

7. Calculate the area of the trapezoid: $h/2 \cdot (f(x_A) + f(x_A + h) - 2y_A)$. Label it *AreaP*, and check Use Label in Custom Tools.

8. Calculate $y_A + AreaP$.

9. Plot the point $(x_A, y_A + AreaP)$ and label this point *B*.

10. Construct segment *AB.*

11. Select point *A*, function *f*, and measurement/parameter *h*, the trapezoid's interior, segment *AB,* and point *B*. Choose **Create New Tool** from **Custom** tools.

12. To automatch function $f(x)$ and *h*, open Script View. Double-click on Given function *f,* and check Automatically Match Sketch Object. Do the same for *h*.

Plot Area with Trapezoids (left)

Activity: "Plotting the Integral"

Document: **PlotIntegral.gsp**

Given: Point *A*, function *f*, and parameter *h*

Result: A trapezoid with width *h* to the left of *A* between the plot of *f* and the *x*-axis, and segment *AB*. The change in *y* between points *A* and *B* is equal to the area of the trapezoid, allowing you to use segment *AB* to create an approximate plot of the integral of $f(x)$.

1. Construct point *A* and measure its *x*- and *y*-coordinates.

2. Calculate $f(x_A)$, $x_A - h$, and $f(x_A - h)$.

3. Plot the points $(x_A, f(x_A))$ and $(x_A - h, f(x_A - h))$.

4. Calculate $x_A - x_A$. (This will give you a value of zero to use in the next step.) Label this *zero,* and check Use Label in Custom Tools.

5. Plot the points $(x_A, zero)$ and $(x_A - h, zero)$.

6. Select these four plotted points, and choose **Construct | Quadrilateral Interior**.

7. Calculate the area of the trapezoid: $h/2 \cdot (f(x_A) + f(x_A - h) - 2y_A)$. Label it *AreaP,* and check Use Label in Custom Tools.

8. Calculate $y_A + AreaP$.

9. Plot the point $(x_A, y_A + AreaP)$ and label this point *B*.

10. Construct segment *AB.*

11. Select point *A*, function *f*, and measurement/parameter *h*, the trapezoid's interior, segment *AB,* and point *B*. Choose **Create New Tool** from **Custom** tools.

12. To automatch function $f(x)$ and *h*, open Script View. Double-click on Given function *f,* and check Automatically Match Sketch Object. Do the same for *h*.