The Slope-Intercept Form of a Line

The slope-intercept form of a line, $y = a + bx$, is one of the best-known formulas in algebra. In this activity you'll learn about this equation first by exploring one line, then by exploring whole *families* of lines.

SKETCH AND INVESTIGATE

Choose **Graph | Define Coordinate System.** To hide the points, select them and choose **Display | Hide Points.**

You'll start this activity with $a = 1$ and $b = 2$ as you explore the line $y = 1 + 2x$.

1. In a new sketch, define a coordinate system and hide the points (0, 0) and (1, 0).

Q1 For $y = 1 + 2x$, what is y when $x = 0$? Write your answer as an ordered pair.

Choose **Graph | Plot Points.** Enter the coordinates in the Plot Points dialog box, click Plot, then click Done.

2. Plot this point. Why does it make sense to call this point the *y-intercept*?

Q2 You found that the y-intercept of $y = 1 + 2x$ is 1. What is the y-intercept of $y = 7 + 3x$? Explain why the y-intercept of $y = a + bx$ is always a.

You've learned that *slope* can be written as *rise/run*. The slope of the line $y = 1 + 2x$ is 2, which you can think of as 2/1 (*rise* = 2 and *run* = 1).

3. Translate your plotted point using this slope. Choose **Transform | Translate,** use a rectangular translation vector, and enter 1 for the run (horizontal) and 2 for the rise (vertical).

To measure the coordinates, choose **Measure | Coordinates.**

Q3 What are the coordinates of the new point? Substitute them into $y = 1 + 2x$ to show they satisfy the equation.

Q4 Translate the new point by the same *rise* and *run* values to get a third point. Find the coordinates of this third point, and verify that it satisfies the equation $y = 1 + 2x$.

4. Select any two of the three points you've plotted, and choose **Construct | Line.**

What you've done so far is one technique for plotting lines in the form $y = a + bx$:

- Plot the y-intercept (0, a).
- Rewrite b as *rise/run* (if necessary).
- Find a second point by translating the y-intercept by *rise* and *run.*
- Connect the points to get the line. Plot a third point to check the line.

If b is a decimal such as 1.5, write it as a fraction such as 3/2. If it's a whole number such as 3, write it as a fraction such as 3/1.

Q5 Using the method just described, plot these lines on graph paper.

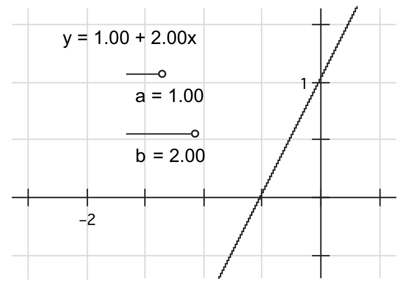
a. $y = -2 + 3x$ b. $y = 2 + (2/3)x$

c. $y = 1 - 2x$ d. $y = -3 + 2.5x$

EXPLORING FAMILIES OF LINES

Now that you've plotted a line, focus on how m and b affect the equation.

5. Open **Slope Intercept 2.gsp.**



The graph of $y = 1 + 2x$ is already plotted. You can change a and b by adjusting their sliders.

To adjust a slider, drag the point at its tip.

Q6 Adjust slider b and observe the effect. Describe the differences between lines with $b > 0$, $b < 0$, and $b = 0$. What happens to the line as b becomes increasingly positive? Increasingly negative?

Q7 Now adjust slider a. Describe the effect this value has on the line.

6. Select the line and choose **Display | Trace Line.**

Q8 Adjust b and observe the trace pattern that forms. Describe the lines that appear when you change b. What do they have in common?

To erase traces left by the line, choose **Display | Erase Traces.**

Q9 Erase the traces and adjust a. How would you describe the lines that form when you change a? What do they have in common?

7. Turn off tracing by selecting the line and choosing **Display | Trace Line** again. Erase any remaining traces.

Q10 For each description below, write the equation in slope-intercept form. To check your equation, adjust a and b so that the line appears on the screen.

a. slope is 2.0; y-intercept is $(0, -3)$

b. slope is -1.5; y-intercept is $(0, 4)$

c. slope is 3.0; x-intercept is $(-2, 0)$

d. slope is -0.4; contains the point $(-6, 2)$

e. contains the points $(3, 5)$ and $(-1, 3)$

EXPLORE MORE

Q11 Attempt to construct a line through the points $(3, 0)$ and $(3, -3)$ by adjusting the sliders in the sketch. Explain why this is impossible. (Why can't you write its equation in slope-intercept form?)

Q12 Can you construct the same line with two different slider configurations? If so, provide two different equations for the same line. If not, explain why.

Objective: Students explore the effects of intercept and slope on the position of a line. They practice writing equations in point-slope form and visualizing the graph when given the equation in point-slope form.

Student Audience: Algebra 1

Prerequisites: Students need to be familiar with the y-intercept and the *rise*/*run* definition of slope.

Sketchpad Level: Intermediate. Students plot and translate points and construct a line.

Activity Time: 25–35 minutes. The second part, Exploring Families of Lines, can be done on a different day, as Q5 is a good stopping point.

Setting: Paired/Individual Activity (use **Slope Intercept2.gsp**) or Whole-Class Presentation (use **Slope Intercept2 Present.gsp**)

The value b is the slope of the line, and a is where the line crosses the y-axis. (This formula can also be written $y = mx + b$, using m and b instead of b and a. Some students may be more familiar with this form.)

SKETCH AND INVESTIGATE

1. Hiding the unit point (0, 1) reduces the chance that students will change the scale of the coordinate system. Sketchpad measures coordinates in graph units but does translation in distance units (usually cm). When the coordinate system is defined, those units agree. If the points in Q3 and Q4 do not have integer values, the student has probably changed the scale by dragging the unit point or the tick numbers on the axes.

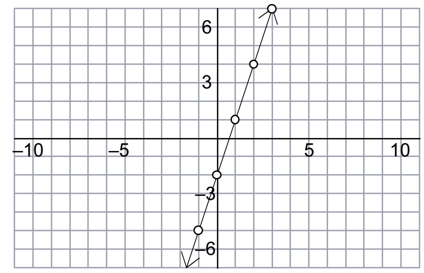
Q1 When $x = 0$, $y = 1$. The point is (0, 1). It makes sense to call this the y-intercept because it's the point where the line crosses the y-axis.

Q2 The y-intercept of $y = 7 + 3x$ is 7. When you substitute 0 for x in $y = a + bx$, you get $y = a + b(0)$, or $y = a$.

Q3 The coordinates of the new point are (1, 3). This satisfies the equation because $y = 1 + 2(1) = 3$. (See the note for step 1 if students get non-integer coordinates when they measure them.)

Q4 The third point is (2, 5). This point satisfies the equation because $y = 1 + 2(2) = 5$.

Q5 The lines are shown below with several integer points plotted.

a.


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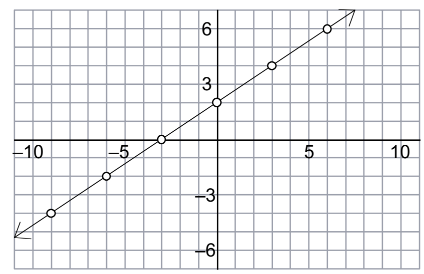
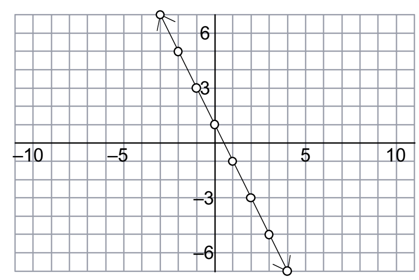
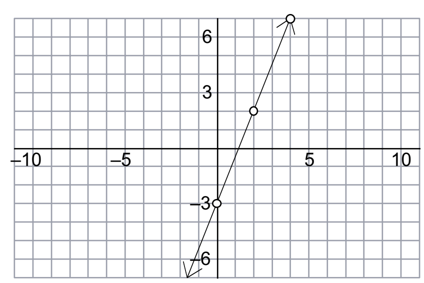


c.


d.


Q6 Lines with a positive b go up to the right and down to the left, lines with a negative b go down to the right and up to the left, and lines with $b = 0$ are horizontal. As b becomes increasingly positive or negative, the line becomes steeper.

Q7 As a becomes increasingly positive, the line is shifted (translated) up. As a becomes increasingly negative, the line is shifted (translated) down. When $a = 0$, the line goes through the origin.

Q8 The slopes vary, but the traces always pass through the same y-intercept. The result looks like an "infinite asterisk."

Q9 This family can be pictured as the infinite set of lines in a plane that are parallel to a given line. They all have the same slope.

Q10 a. $y = -3 + 2x$ b. $y = 4 - 1.5x$

c. $y = 6 + 3x$ d. $y = -0.4 - 0.4x$

e. $y = 3.5 + 0.5x$

EXPLORE MORE

Q11 This line is parallel to the y-axis, so it has no y-intercept and the slope is undefined. The line can be expressed with the equation $x = 3$, but that's not in slope-intercept form.

Q12 No, it's not possible. The reason is that every line has a unique y-intercept, so there's only one value for a for a particular line. Similarly, each line has a unique slope, so there's only one value for b.

WHOLE-CLASS PRESENTATION

Use the sketch **Slope Intercept2 Present.gsp** to help students visualize the graph of a line from an equation written in slope-intercept form. You will need to discuss how the y-intercept is found by substituting 0 for x, which always yields $y = a$ for an equation in the form $y = a + bx$. Then the slope can be applied to find one or two more points and graph the line.

Use page 2 to further explore the effects of a and b. This sketch is set up with sliders for a and b. You can use this sketch to explore Q6–Q12 with the whole class.