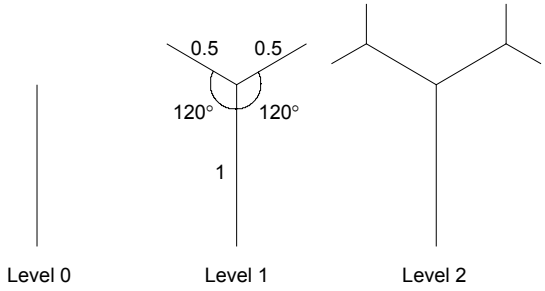
More Fractals

In *Exploring Algebra 1 with The Geometer's Sketchpad* there are several activities that give you a chance to generate fractals and explore the related concepts of self-similarity and ratio of similarity. In this activity you will move on to more complex fractals. Aside from the mathematical concepts, many fractals create striking visual patterns—have some fun with this.

MULTIPLE MAPPINGS

To make a fractal grow really quickly, you need to define more than one mapping. Try that with this branching fractal, which has two mappings.

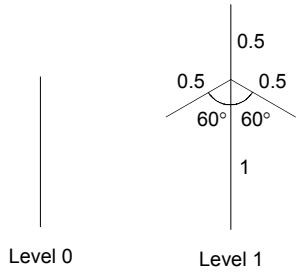


1. Open **More Fractals.gsp.** Select in order point *A*, point *B*, and parameter *depth*. Holding the Shift key, choose **Transform | Iterate To Depth.**
2. In the dialog box, map *A* to *B* and map *B* to *C*. From the Structure pop-up menu, choose **Add New Map**. Now map *A* to *B* and map *B* to *D*. Choose Iterate.

For all of the fractals in this activity, assume the length of level 0 to be 1.

Q1 Change the *depth* parameter to see higher levels. What are the total lengths at levels 1, 2, and 3?

Q2 Change the ratio. At what ratio do the branches begin to cross?

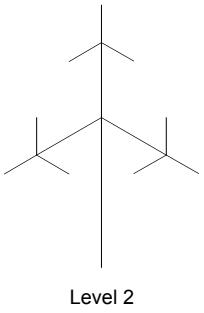


Now try this fractal with three branches.

3. On page 2 follow the same procedure to create these three mappings:

Pre-Image	To	Map #3	Map #2	Map #1
A	⇒	*B*	*B*	*B*
B	⇒	*E*	*D*	*C*

Q3 Compute the total length for the first three levels and state whether it crosses itself.



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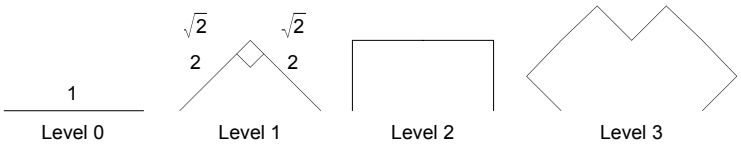
FINAL ITERATION

With some fractals, the most interesting form appears when you see only the objects that were added on the last iteration. This one begins as a line segment. The first level is a right angle.



4. On page 3 create two mappings by following this table.

Pre-Image	To	Map #2	Map #1
A	⇒	*C*	*A*
B	⇒	*B*	*C*

Did you neglect to do step 5 before closing the dialog box? Select the iterated image, choose **Edit | Properties | Iteration**, and set Final Iteration Only.

5. Before closing the Iterate dialog box, use the Display pop-up menu to choose **Final Iteration Only**. Then click **Iterate**.
6. Press the *Hide Level 0* button to hide the original line segment.

Q4 Compute the total length for the first three levels. Count only the lengths of the segments that are actually shown.

Q5 Pages 4 and 5 have more final-iteration fractals. The instructions are on the sketches. Complete the constructions and compute the length for the first three levels.

EXPLORE OPEN FRACTALS

The remaining pages contain some *open* fractals. Rather than being restricted to a predetermined shape and ratio, these constructions allow you to alter the shape after you have constructed the iteration. In each case level 0 of the fractal appears in black. Follow the directions in the table in order to make multiple mappings. After constructing the fractal, increase the depth and drag the points on the red figure to modify the fractal definition.

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Objective: Students use iterative constructions to build fractals with multiple mappings.

Student Audience: Algebra 1/Algebra 2, Geometry

Prerequisites: Students should have experience constructing simpler fractals, such as those in the activities Fractals and Length of the Koch Curve.

Sketchpad Level: Intermediate. All of the fractals are prepared in advance, but students must execute some more advanced iteration commands.

Class Time: 30–40 minutes. The activity may be abbreviated or spread over more than one session.

Setting: Paired/Individual Activity (use **More Fractals.gsp**)

MULTIPLE MAPPINGS

In these constructions the number of new objects introduced by each iteration increases exponentially. Computers will slow down or even lock up if the *depth* parameter is made too large.

Q1

Level	1	2	3
Length	2	3	4

Q2 The limiting ratio at which the branches begin to cross is about 0.62. The exact value is the golden ratio:

$$\frac{\left(\sqrt{5}-1\right)}{2}$$

Q3

Level	1	2	3
Length	5/2	19/4	65/8

Q4

Level	1	2	3
Length	$\sqrt{2}$	2	$2\sqrt{2}$

Q5 These two fractal lengths both increase by geometric progressions. Students will likely have trouble finding the ratio of the progression. Show them how to get the fractal length by making measurements. For example, on the page 4 meander fractal, they can measure *AC*, *CD*, and *DB*, and compare those lengths to *AB*. The short segments are each one-half the length of *AB*. Therefore the length of level 1 must be 3/2.

Page 4

Level	1	2	3
Length	3/2	9/4	27/8

Page 5

Level	1	2	3
Length	2	4	8

This last one has the interesting property of filling an area. The square that is filled does not have the same alignment as the squares that make up the fractal.

EXPLORE OPEN FRACTALS

This section is mostly for fun. Since the fractal definition depends on the free points in the red figure, there is no fixed ratio of similarity, so the fractal length calculations are not possible.

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