The Point-Slope Form of a Line

The slope-intercept form of a line is great if you know one special point: the y-intercept. But what if the point you know is an everyday, ordinary point such as $(3, -2)$ or $(-7, -7)$? In this case it's usually most convenient to use the *point-slope form* of a line, which you'll study in this activity.

SKETCH AND INVESTIGATE

1. Open the sketch **Point Slope2.gsp.**

To adjust a slider, drag the point at its tip.

You'll see an equation in the point-slope form $y = y_1 + b(x - x_1)$, with numbers filled in for b, x_1, and y_1. Adjust the sliders for b, x_1, and y_1, and watch the equation change. There's no line yet, but you can graph one.

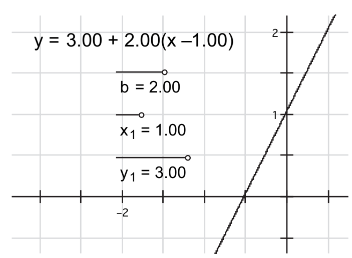
2. Choose **Graph | Plot New Function.**

The New Function dialog box appears.

To enter b, x_1, and y_1, click their measurements in the sketch. To enter x, click the x key on the keypad.

3. Enter $y_1 + b(x - x_1)$ and click OK.

Sketchpad plots the function for the current values of the parameters b, x_1, and y_1.



4. Select the new line and choose **Display | Trace Function Plot.**

If at any point you wish to erase traces left by the line, choose **Display | Erase Traces.**

Q1 Adjust slider b. You'll see that the line rotates around a single point. Change the values of x_1 and y_1, then adjust b again, focusing on where this point appears to be. What are the point's coordinates? How do they relate to x_1 and y_1?

To deselect all objects, click the **Arrow** tool in empty space.

5. Deselect all objects. Now select, in order, measurement x_1 and measurement y_1. Choose **Graph | Plot As (x, y)** to plot the point (x_1, y_1).

Q2 Adjust slider b again and observe what happens. Does this support your answer from Q1?

Q3 Describe the family of lines that forms when you change b.

Q4 Adjust sliders x_1 and y_1, one at a time. How would you describe the families of lines that form when varying each of these values? How do they compare to each other?

Q5 Summarize the roles that x_1 and y_1 play in the equation $y = y_1 + b(x - x_1)$.

Q6 Suppose you know that the slope of a line is 2 and that it contains the point $(1, 3)$. What is the equation in point-slope form for this line? Check your answer by adjusting the sliders in the sketch.

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Q7 Write an equation in point-slope form for each of the lines described. When you finish, check each of your answers by adjusting sliders b, x_1, and y_1 so that the line is drawn on the screen.

a. slope is 2; contains the point $(-2, 1)$

b. slope is -1; contains the point $(-2, 1)$

c. is parallel to the line $y = 4 + 3(x - 2)$; goes through the origin

d. slope is $\frac{4}{5}$; x-intercept is $(2, 0)$

e. contains the points $(2, 3)$ and $(-1, 4)$

f. contains the points $(-3, 5)$ and $(4, 5)$

EXPLORE MORE

Q8 Try to construct a line through the points $(2, 3)$ and $(2, -2)$ by adjusting the sliders in the sketch. Explain why this is impossible and why this equation cannot be written in point-slope form.

Q9 Is it possible to construct the same line with different slider configurations? If not, explain why. If so, provide two different equations for the same line.

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Objective: Students use Sketchpad's dynamic capabilities to examine the effect of each constant on a linear equation written in point-slope form. This activity helps to demystify the point-slope form.

Audience: Algebra 1

Prerequisites: Students should already have studied the slope-intercept form of a line.

Sketchpad Level: Easy/Intermediate. The hardest step is step 3, in which students plot the function (see step 3 below).

Activity Time: 25–35 minutes

Setting: Paired/Individual Activity (use **Point Slope2.gsp**) or Whole-Class Presentation (use **Point Slope2 Present.gsp**)

SKETCH AND INVESTIGATE

3. Students may have to drag the New Function dialog box by its title bar in order to see measurements y_1, b, and x_1 in the sketch. Students will need to enter the (implied) multiplication sign after b.

Q1 The line appears to spin around the point (x_1, y_1).

Q2 This does support the answer from Q1. You can now see the center point as the line spins.

Q3 This is the family of lines through the point (x_1, y_1) with any slope. This family can be pictured as an asterisk with the center point (x_1, y_1).

Q4 These are families of lines with the same slope. The families can be pictured as the infinite set of lines in a plane parallel to a given line. It's interesting that although the two families look the same, they are formed in different ways, as you can see by watching the point (x_1, y_1). Adjusting x_1 moves the lines right and left whereas adjusting y_1 moves the lines up and down.

Q5 Parameter x_1 is the x-coordinate of the special point that the line spins around when b is dragged. Making x_1 larger moves the line to the right; making it smaller (or more negative) moves it to the left.

Parameter y_1 is the y-coordinate of the special point that the line spins around when b is dragged. Making y_1 larger moves the line up; making it smaller (more negative) moves it down.

Q6 $y = 3 + 2(x - 1)$

Q7
a. $y = 1 + 2(x - (-2))$ or $y = 1 + 2(x + 2)$
b. $y = 1 - 1(x - (-2))$ or $y = 1 - (x + 2)$
c. $y = 0 + 3(x - 0)$ or $y = 3x$
d. $y = 0 + 0.8(x - 2)$ or $y = 0.8(x - 2)$
e. $y = 3 - (1/3)(x - 2)$ or $y = 4 - (1/3)(x + 1)$
f. $y = 5 + 0(x - 4)$ or $y = 5$

EXPLORE MORE

Q8 Slope is defined as *rise*/*run*. For this line, since the x-coordinates of both points are the same (2), $run = 0$. Thus the slope is undefined, since you can't divide by 0. The line can be expressed with the equation $x = 2$, but that's not in point-slope form.

Q9 It is possible to express any line (except vertical lines) with an infinite number of equations in point-slope form (or an infinite number of slider configurations in the sketch). The reason is that x_1 and y_1 aren't unique—any point on the line will do. For example, the line $y = 3 + 2(x - 1)$ also goes through the points (2, 5) and (3, 7), so the equations $y = 5 + 2(x - 2)$ and $y = 7 + 2(x - 3)$ also express this same line. (Try it out!)

WHOLE-CLASS PRESENTATION

Students see the effects of changing each of the values x_1, y_1, and b. They see how changing these values in a linear equation in point-slope form changes the location of the line on the graph. Use the sketch **Point Slope2 Present.gsp.** Press the button *Show Point on Line,* and drag x_p to show how the calculated y value changes for different values of x. Then press the button *Show Function and Line,* and drag x_p again. Use the sliders to change x_1, y_1, and b and let students make observations. Do Q1–Q7 together as a class.

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